ON THE STABILITY OF INELASTIC PLATES, TAKING TRANSVERSE SHEAR DEFORMATIONS INTO ACCOUNT

(OB USTOICHIVOSTI NEUPRUGIKH PLASTINOK S UCHETOM DEFORMATSII POPERECHNYKH SDVIGOV)

PMM Vol.27, No.4, 1963, pp. 753-757

S.A. AMBARTSUMIAN
(Erevan)

(Received January 29, 1962)

We will investigate the problem of buckling of inelastic plates under the condition of steady loading, taking into account effects connected with transverse deformations of the plate. The problem will be solved with the aid of the theory of plastic flow. The analogous problem, without consideration of transverse shear, has been treated by Kachanov [1].

1. Together with the basic assumptions of the theory of plasticity, we will adopt:

a) the hypothesis of continuous loading [1-3] according to which bending of the plate is possible under conditions of increase of load, which provides loading at all points of the plate and;

b) the theory of plates without the hypothesis that normals remain undeformed [4]. Accordingly, it will be assumed that approximately*

$$\tau_{xz} = f(z)\,\varphi(x, y), \quad e_z = 0, \qquad \tau_{yz} = f(z)\,\psi(x, y), \quad \sigma_z = 0 \qquad (f(\pm{}^1/_2 h) = 0) \tag{1.1}$$

where $f(z)$ is a prescribed law of variation of the shearing stresses τ_{xz}, τ_{yz} through the thickness of the plate; $\varphi(x, y)$, $\psi(x, y)$ are functions to be determined. The plate has been represented in a system of Cartesian coordinates x, y, z with the z-axis normal to the middle surface.

---

* Here and henceforth, the familiar notation [1,4] will be used.

2. Let the plate be strained to the elastic limit by the momentless state of stress

$$\sigma_x = -p, \qquad \sigma_y = -q, \qquad \tau_{xy} = r \tag{2.1}$$

During buckling, the stresses in the plate receive the infinitesimal increments $\delta\sigma_x$, $\delta\sigma_y$, $\delta\tau_{xy}$, $\delta\tau_{xz}$ and $\delta\tau_{yz}$. The components of strain also receive infinitesimal increments. In the general case of a plate composed of a work-hardening material, we can write [1]

$$\begin{gathered}\delta\varepsilon_x = \frac{1}{E}(\delta\sigma_x - \nu\delta\sigma_y) - \frac{1}{3}F(T)\,\delta T\,(2p-q),\\ \delta\varepsilon_y = \frac{1}{E}(\delta\sigma_y - \nu\sigma\delta_x) - \frac{1}{3}F(T)\,\delta T\,(2q-p)\\ \delta\gamma_{xy} = \frac{1}{G}\delta\tau_{xy} + 2F(T)\,r\,\delta T, \qquad \delta\gamma_{xz} = \frac{1}{G}\delta\tau_{xz}, \qquad \delta\gamma_{yz} = \frac{1}{G}\delta\tau_{yz}\end{gathered} \tag{2.2}$$

where

$$\begin{gathered}F(T) = \frac{1}{2T^2}\frac{d\Phi}{dT}, \qquad T^2 = \frac{p^2 - pq + q^2 + 3r^2}{3}\\ \delta T = -\frac{1}{6T}\left[(2p-q)\,\delta\sigma_x + (2q-p)\,\delta\sigma_y - 6r\,\delta\tau_{xy}\right]\end{gathered} \tag{2.3}$$

and $\Phi(T)$ is a characteristic function of the stress intensity T for the given material, not depending on the nature of the state of stress.

By virtue of (2.3) and (2.2), one easily obtains

$$\begin{gathered}E\delta\varepsilon_x = A_{11}\delta\sigma_x + A_{12}\delta\sigma_y + A_{16}\delta\tau_{xy}, \quad E\delta\varepsilon_y = A_{22}\delta\sigma_y + A_{12}\delta\sigma_x + A_{26}\delta\tau_{xy}\\ E\delta\gamma_{xy} = A_{66}\delta\tau_{xy} + A_{16}\delta\sigma_x + A_{26}\delta\sigma_y, \quad E\delta\gamma_{xz} = A_{55}\delta\tau_{xz}, \qquad E\delta\tau_{yz} = A_{44}\delta\tau_{yz}\end{gathered} \tag{2.4}$$

where

$$\begin{gathered}A_{11} = 1 + \theta\,(2p-q)^2, \qquad A_{12} = \theta\,(2q-p)(2p-q) - \nu, \quad A_{22} = 1 + \theta\,(2q-p)^2\\ A_{66} = \frac{E}{G} + 36\theta\,r^2, \qquad A_{44} = A_{55} = \frac{E}{G'} \qquad \left(\theta = \frac{E}{18}\frac{F(T)}{T}\right)\\ A_{16} = -6\theta\,r\,(2p-q), \qquad A_{26} = -6\theta\,r\,(2q-p)\end{gathered} \tag{2.5}$$

Solving (2.4) for the respective stress increments, we obtain

$$\begin{gathered}\delta\sigma_x = a_{11}\delta\varepsilon_x + a_{12}\delta\varepsilon_y + a_{16}\delta\gamma_{xy}, \qquad \delta\sigma_y = a_{22}\delta\varepsilon_y + a_{12}\delta\varepsilon_x + a_{26}\,\delta\gamma_{xy}\\ \delta\tau_{xy} = a_{66}\delta\gamma_{xy} + a_{16}\delta\varepsilon_x + a_{26}\delta\varepsilon_y, \qquad \delta\tau_{xz} = a_{55}\delta\gamma_{xz}, \qquad \delta\tau_{yz} = a_{44}\delta\gamma_{yz}\end{gathered} \tag{2.6}$$

where

$$\begin{gathered}a_{11} = \frac{E}{\Omega}(A_{22}A_{66} - A_{26}^2), \qquad a_{22} = \frac{E}{\Omega}(A_{11}A_{66} - A_{16}^2), \qquad a_{12} = \frac{E}{\Omega}(A_{16}A_{26} - A_{12}A_{66})\\ a_{66} = \frac{E}{\Omega}(A_{11}A_{22} - A_{12}^2), \qquad a_{16} = \frac{E}{\Omega}(A_{12}A_{26} - A_{22}A_{16}), \qquad a_{26} = \frac{E}{\Omega}(A_{12}A_{16} - A_{11}A_{26})\end{gathered} \tag{2.7}$$

$$a_{44} = G,\ a_{55} = G,\ \Omega = A_{66}\left[A_{11}A_{22} - A_{12}^2 + \frac{2A_{12}A_{16}A_{26} - A_{11}A_{26}^2 - A_{22}A_{16}^2}{A_{66}}\right]$$

By virtue of (1.1) and (2.6), for the increments of transverse shear strain we obtain

$$\delta\gamma_{xz} = \frac{1}{a_{55}} f(z)\ \varphi(x, y), \quad \delta\gamma_{yz} = \frac{1}{a_{44}} f(z)\,\psi(x, y) \tag{2.8}$$

where $\varphi(x, y)$ and $\psi(x, y)$ are yet unknown functions characterizing the infinitesimal increments of transverse shear.

According to flexure theory for plates without the hypothesis that normals remain undeformed, the displacements accompanying buckling will be

$$u_x = u^\circ + \delta u - z\frac{\partial w}{\partial x} + \frac{J_0}{a_{55}}\varphi, \qquad u_y = v^\circ + \delta v - z\frac{\partial w}{\partial y} + \frac{J_0}{a_{44}}\psi \qquad \left(J_0 = \int_0^z f(z)\,dz\right) \tag{2.9}$$

where $u^\circ = u^\circ(x, y)$ and $v^\circ = v^\circ(x, y)$ are the tangential displacements of the middle surface resulting from the momentless state of stress (2.1); $\delta u = \delta u(x, y)$, $\delta v = \delta v(x, y)$ and $w = w(x, y)$ are the increments of tangential and normal displacement of the middle surface of the plate due to buckling.

In view of (2.9), for the strain increments due to buckling we obtain

$$\delta\varepsilon_x = \frac{\partial\delta u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{J_0}{a_{55}}\frac{\partial\varphi}{\partial x}, \qquad \delta\varepsilon_y = \frac{\partial\delta v}{\partial y} - z\frac{\partial^2 w}{\partial y^2} + \frac{J_0}{a_{44}}\frac{\partial\psi}{\partial y}$$

$$\delta\gamma_{xy} = \frac{\partial\delta u}{\partial y} + \frac{\partial\delta v}{\partial x} - 2z\frac{\partial^2 w}{\partial x\,\partial y} + J_0\left(\frac{1}{a_{55}}\frac{\partial\varphi}{\partial y} + \frac{1}{a_{44}}\frac{\partial\psi}{\partial x}\right)$$

Substituting into (2.6) the expressions for the strain increments we obtain the following increments of stress:

$$\delta\sigma_x = -z\left(a_{11}\frac{\partial^2 w}{\partial x^2} + a_{12}\frac{\partial^2 w}{\partial y^2} + 2a_{16}\frac{\partial^2 w}{\partial x\,\partial y}\right) + J_0\left(\frac{a_{11}}{a_{55}}\frac{\partial\varphi}{\partial x} + \frac{a_{12}}{a_{44}}\frac{\partial\psi}{\partial y} + \right.$$
$$\left. + \frac{a_{16}}{a_{55}}\frac{\partial\varphi}{\partial y} + \frac{a_{16}}{a_{44}}\frac{\partial\psi}{\partial x}\right) + a_{11}\frac{\partial\delta u}{\partial x} + a_{12}\frac{\partial\delta v}{\partial y} + a_{16}\left(\frac{\partial\delta u}{\partial y} + \frac{\partial\delta v}{\partial x}\right)$$

$$\delta\sigma_y = -z\left(a_{22}\frac{\partial^2 w}{\partial y^2} + a_{12}\frac{\partial^2 w}{\partial x^2} + 2a_{26}\frac{\partial^2 w}{\partial x\,\partial y}\right) + J_0\left(\frac{a_{22}}{a_{44}}\frac{\partial\psi}{\partial y} + \frac{a_{12}}{a_{55}}\frac{\partial\varphi}{\partial x} + \right.$$
$$\left. + \frac{a_{26}}{a_{44}}\frac{\partial\psi}{\partial x} + \frac{a_{26}}{a_{55}}\frac{\partial\varphi}{\partial y}\right) + a_{22}\frac{\partial\delta v}{\partial y} + a_{12}\frac{\partial\delta u}{\partial x} + a_{26}\left(\frac{\partial\delta v}{\partial x} + \frac{\partial\delta u}{\partial y}\right)$$

$$\delta\tau_{xy} = -z\left(a_{16}\frac{\partial^2 w}{\partial x^2} + a_{26}\frac{\partial^2 w}{\partial y^2} + 2a_{66}\frac{\partial^2 w}{\partial x\,\partial y}\right) + J_0\left(\frac{a_{16}}{a_{55}}\frac{\partial\varphi}{\partial x} + \frac{a_{26}}{a_{44}}\frac{\partial\psi}{\partial y} + \right.$$
$$\left. + \frac{a_{66}}{a_{55}}\frac{\partial\varphi}{\partial y} + \frac{a_{66}}{a_{44}}\frac{\partial\psi}{\partial x}\right) + a_{16}\frac{\partial\delta u}{\partial x} + a_{26}\frac{\partial\delta v}{\partial y} + a_{66}\left(\frac{\partial\delta u}{\partial y} + \frac{\partial\delta v}{\partial x}\right)$$

$$\delta\tau_{xz} = f(z)\,\varphi, \qquad \delta\tau_{yz} = f(z)\,\psi$$

Proceeding in the usual way, it is easy to calculate the increments of the bending moments and shearing forces

$$\delta M_x = -\frac{h^3}{12}\left(a_{11}\frac{\partial^2 w}{\partial x^2} + a_{12}\frac{\partial^2 w}{\partial y^2} + 2a_{16}\frac{\partial^2 w}{\partial x\,\partial y}\right) + $$
$$+ J_1\left[\frac{1}{a_{55}}\left(a_{11}\frac{\partial \varphi}{\partial x} + a_{16}\frac{\partial \varphi}{\partial y}\right) + \frac{1}{a_{44}}\left(a_{12}\frac{\partial \psi}{\partial y} + a_{16}\frac{\partial \psi}{\partial x}\right)\right]$$

$$\delta M_y = -\frac{h^3}{12}\left(a_{22}\frac{\partial^2 w}{\partial y^2} + a_{12}\frac{\partial^2 w}{\partial x^2} + 2a_{26}\frac{\partial^2 w}{\partial x\,\partial y}\right) + J_1\left[\frac{1}{a_{55}}\left(a_{12}\frac{\partial \varphi}{\partial x} + a_{26}\frac{\partial \varphi}{\partial y}\right) + \right.$$
$$\left. + \frac{1}{a_{44}}\left(a_{22}\frac{\partial \psi}{\partial y} + a_{26}\frac{\partial \psi}{\partial x}\right)\right] \qquad (2.10)$$

$$\delta H = -\frac{h^3}{12}\left(2a_{66}\frac{\partial^2 w}{\partial x\,\partial y} + a_{16}\frac{\partial^2 w}{\partial x^2} + a_{26}\frac{\partial^2 w}{\partial y^2}\right) + J_1\left[\frac{1}{a_{55}}\left(a_{16}\frac{\partial \varphi}{\partial x} + a_{66}\frac{\partial \varphi}{\partial y}\right) + \right.$$
$$\left. + \frac{1}{a_{44}}\left(a_{26}\frac{\partial \psi}{\partial y} + a_{66}\frac{\partial \psi}{\partial x}\right)\right]$$

$$\delta N_1 = J_2\varphi, \qquad \delta N_2 = J_2\psi, \qquad J_1 = \int_{-h/2}^{h/2} zJ_0(z)\,dz, \qquad J_2 = \int_{-h/2}^{h/2} f(z)\,dz \qquad (2.11)$$

3. The equations of equilibrium of an infinitesimal element of the plate after buckling will have the form

$$\frac{\partial \delta M_x}{\partial x} + \frac{\partial \delta H}{\partial y} = \delta N_1, \qquad \frac{\partial \delta M_y}{\partial y} + \frac{\partial \delta H}{\partial x} = \delta N_2 \qquad (3.1)$$

$$\frac{\partial \delta N_1}{\partial x} + \frac{\partial \delta N_2}{\partial y} + T_1^\circ\frac{\partial^2 w}{\partial x^2} + T_2^\circ\frac{\partial^2 w}{\partial y^2} + 2S^\circ\frac{\partial^2 w}{\partial x\,\partial y} = 0 \qquad (3.2)$$

where T_1°, T_2°, S° are the internal tangential forces of the initial momentless state, i.e. $T_1^\circ = -ph$, $T_2^\circ = -qh$, $S^\circ = rh$.

Substituting the values of the increments of the internal forces and moments into the equations of equilibrium (3.1) and (3.2), we obtain for the three functions $w(x, y)$, $\varphi(x, y)$ and $\psi(x, y)$ the system

$$\frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y} = \frac{h}{J_2}\left(p\frac{\partial^2 w}{\partial x^2} + q\frac{\partial^2 w}{\partial y^2} - 2r\frac{\partial^2 w}{\partial x\,\partial y}\right) \qquad (3.3)$$

$$a_{11}\frac{\partial^3 w}{\partial x^3} + (a_{12} + 2a_{66})\frac{\partial^3 w}{\partial x\,\partial y^2} + 3a_{16}\frac{\partial^3 w}{\partial x^2\,\partial y} + a_{26}\frac{\partial^3 w}{\partial y^3} - \frac{12}{h^3}J_1\left[\frac{1}{a_{55}}\left(a_{11}\frac{\partial^2 \varphi}{\partial x^2} + \right.\right.$$
$$\left.\left. + 2a_{16}\frac{\partial^2 \varphi}{\partial x\,\partial y} + a_{66}\frac{\partial^2 \varphi}{\partial y^2}\right) + \frac{1}{a_{44}}\left(a_{16}\frac{\partial^2 \psi}{\partial x^2} + (a_{12} + a_{66})\frac{\partial^2 \psi}{\partial x\,\partial y} + a_{26}\frac{\partial^2 \psi}{\partial y^2}\right)\right] + \frac{12}{h^3}J_2\varphi = 0 \qquad (3.4)$$

$$a_{22}\frac{\partial^3 w}{\partial y^3} + (a_{12} + 2a_{66})\frac{\partial^3 w}{\partial x^2\,\partial y} + 3a_{26}\frac{\partial^3 w}{\partial x\,\partial y^2} + a_{16}\frac{\partial^3 w}{\partial x^3} - \frac{12}{h^3}J_1\left[\frac{1}{a_{44}}\left(a_{22}\frac{\partial^2 \psi}{\partial y^2} + \right.\right.$$
$$\left.\left. + 2a_{26}\frac{\partial^2 \psi}{\partial x\,\partial y} + a_{66}\frac{\partial^2 \psi}{\partial x^2}\right) + \frac{1}{a_{55}}\left(a_{26}\frac{\partial^2 \varphi}{\partial y^2} + (a_{12} + a_{66})\frac{\partial^2 \varphi}{\partial x\,\partial y} + a_{16}\frac{\partial^2 \varphi}{\partial x^2}\right)\right] + \frac{12}{h^3}J_2\psi = 0 \qquad (3.5)$$

By introducing one function $\Phi(x, y)$, the system of equations (3.3) to (3.5) can be reduced to a single equation of sixth order. Setting

$$w=\frac{144}{h^6}\left[J_2^2-J_1J_2\left(\frac{a_{11}}{a_{55}}+\frac{a_{66}}{a_{44}}\right)\frac{\partial^2}{\partial x^2}-2J_1J_2\left(\frac{a_{16}}{a_{55}}+\frac{a_{26}}{a_{44}}\right)\frac{\partial^2}{\partial x\,\partial y}+\right.$$
$$\left.+J_1J_2\left(\frac{a_{22}}{a_{44}}+\frac{a_{66}}{a_{55}}\right)\frac{\partial^2}{\partial y^2}+\frac{J_1^2}{a_{44}a_{55}}L(a_{ik})\right]\Phi \tag{3.6}$$

$$\varphi=\frac{12}{h^3}\left\{-J_2\left[a_{11}\frac{\partial^3}{\partial x^3}+(a_{12}+2a_{66})\frac{\partial^3}{\partial x\,\partial y^2}+3a_{16}\frac{\partial^3}{\partial x^2\,\partial y}+a_{26}\frac{\partial^3}{\partial y^3}\right]+\frac{J_1}{a_{44}}\frac{\partial}{\partial x}L(a_{ik})\right\}\Phi$$
$$\psi=\frac{12}{h^3}\left\{-J_2\left[a_{22}\frac{\partial^3}{\partial y^3}+(a_{12}+2a_{66})\frac{\partial^3}{\partial x^2\,\partial y}+3a_{26}\frac{\partial^3}{\partial x\,\partial y^2}+a_{16}\frac{\partial^3}{\partial x^3}\right]+\frac{J_1}{a_{55}}\frac{\partial}{\partial y}L(a_{ik})\right\}\Phi \tag{3.7}$$

$$L(a_{ik})=c_{11}\frac{\partial^4}{\partial x^4}+2c_{16}\frac{\partial^4}{\partial x^3\partial y}+c_{12}\frac{\partial^4}{\partial x^2\,\partial y^2}+2c_{26}\frac{\partial^4}{\partial x\,\partial y^3}+c_{22}\frac{\partial^4}{\partial y^4}$$

$$c_{11}=a_{11}a_{66}-a_{16}^2,\qquad c_{16}=a_{11}a_{26}-a_{12}a_{16},\qquad c_{22}=a_{22}a_{66}-a_{26}^2$$
$$c_{26}=a_{22}a_{16}-a_{12}a_{26},\qquad c_{12}=a_{11}a_{22}-a_{12}^2-2(a_{12}a_{66}-a_{16}a_{26}) \tag{3.8}$$

equations (3.4) and (3.5) are satisfied identically and from (3.3) we obtain the resolving equation of the problem

$$a_{11}\frac{\partial^4\Phi}{\partial x^4}+4a_{16}\frac{\partial^4\Phi}{\partial x^3\,\partial y}+2(a_{12}+2a_{66})\frac{\partial^4\Phi}{\partial x^2\,\partial y^2}+4a_{26}\frac{\partial^4\Phi}{\partial x\,\partial y^3}+a_{22}\frac{\partial^4\Phi}{\partial y^4}-$$
$$-\frac{J_1}{J_2}\left[\frac{c_{11}}{a_{44}}\frac{\partial^6\Phi}{\partial x^6}+2\frac{c_{16}}{a_{44}}\frac{\partial^6\Phi}{\partial x^5\,\partial y}+\left(\frac{c_{12}}{a_{44}}+\frac{c_{11}}{a_{55}}\right)\frac{\partial^6\Phi}{\partial x^4\,\partial y^2}+2\left(\frac{c_{26}}{a_{44}}+\frac{c_{16}}{a_{55}}\right)\frac{\partial^6\Phi}{\partial x^3\,\partial y^3}+\right.$$
$$\left.+\left(\frac{c_{12}}{a_{55}}+\frac{c_{22}}{a_{44}}\right)\frac{\partial^6\Phi}{\partial x^2\,\partial y^4}+2\frac{c_{26}}{a_{55}}\frac{\partial^6\Phi}{\partial x\,\partial y^5}+\frac{c_{22}}{a_{55}}\frac{\partial^6\Phi}{\partial y^6}\right]=$$
$$=\frac{h^3}{12J_2^2}\left(T_1^\circ\frac{\partial^2 w}{\partial x^2}+T_2^\circ\frac{\partial^2 w}{\partial y^2}+2S^\circ\frac{\partial^2 w}{\partial x\,\partial y}\right) \tag{3.9}$$

In studying the system of equations (3.3) to (3.5) and the equation (3.9), it is easily noted that they are superficially similar to the corresponding partial equations of the theory of anisotropic plates formed without the hypothesis of undeformable normals [4].

4. We will study the problem of the stability of a column [1]. Let a straight rectangular column be supported by two straight sides ($x = 0$, $x = l$) and be compressed by a uniformly distributed pressure (along these same edges) of intensity p ($q = 0$, $r = 0$). Let the column be so straight that it can assume a cylindrical shape in the case of instability, i.e. $w = w(x)$.

In view of the initial state of the present problem, the differential equation of stability takes the form

$$a_{11}\frac{h^3}{12}\frac{d^4w}{dx^4}=-ph\frac{d^2w}{dx^2}+ph\frac{a_{11}}{a_{55}}\frac{J_1}{J_2}\frac{d^4w}{dx^4} \tag{4.1}$$

By assuming $w = B\sin(m\pi x/l)$ we satisfy the end conditions, and for the critical pressure we obtain

$$p = a_{11}\frac{h^3}{12}\frac{m^2\pi^2}{l^2 h}\left(1 + \frac{m^2\pi^2}{l^2}\frac{a_{11}}{a_{55}}\frac{J_1}{J_2}\right)^{-1} \tag{4.2}$$

On the strength of (2.5) and the initial conditions for the problem, from (2.7) we obtain for the coefficients a_{11} and a_{55} the values

$$a_{11} = E\frac{1 + \theta p^2}{1 - \nu^2 + \theta p^2(5-4\nu)}, \qquad a_{55} = G$$

The value θ defined in (2.5) is the same as that in a previously treated similar problem [1], and can be determined from a condition of simple tension (or compression). In the case of simple tension, assuming $\varepsilon_x = f(\sigma_x)$ and taking account of the fact that during work hardening the work of plastic deformation A does not depend on the loading path and is simply a function of the loading intensity, i.e. $A = \Phi(T)$, we obtain by means of simple transformation [1]

$$\theta = \frac{1}{12T^2}\left(\frac{E}{E'} - 1\right) = \frac{1}{4p^2}\left(\frac{E}{E'} - 1\right) \tag{4.3}$$

where E is the elastic modulus, and E' is the local or tangent modulus.

Assuming $f(z) = (1/4\ h^2 - z^2)$ for the calculation of the ratio J_1/J_2, on the strength of (2.11) we obtain $J_1/J_2 = 1/10\ h^2$.

Substitution of the expressions θ and J_1/J_2 into (4.2), we finally find for the critical pressure

$$P_* = P_*^\circ K\left[1 + \frac{m^2\pi^2}{10}\frac{h^2}{l^2}\frac{E}{G(1-\nu^2)}K\right]^{-1}, \qquad P_*^\circ = \frac{m^2\pi^2}{l^2}\frac{D}{h}$$

$$K = \left[1 + \frac{1}{4}\left(\frac{E}{E'} - 1\right)\right]\left[1 + \frac{5-4\nu}{4(1-\nu^2)}\left(\frac{E}{E'} - 1\right)\right]^{-1}, \qquad D = \frac{Eh^3}{12(1-\nu^2)}$$

where P_*° is the value of the critical pressure for an elastic column found without consideration of the effects of transverse shears.

In particular, neglecting the correction for the consideration of the transverse shear strains, we obtain the critical pressure found in [1], namely

$$P_* = P_*^\circ K$$

A few words about the function $f(z)$. As is well-known [5,6], on the yield surface the diagram of the shearing stresses τ_{xz} degenerates into a triangular shape, in connection with which there arises the question as to the validity of the assumed parabolic law of distribution of shearing stresses.

Let us assume that

$$f(z) = e - (2e / h) z \operatorname{sign} z$$

From (2.11) we obtain $J_1/J_2 = 10\ h^2/96$. As is to be expected [7], the value of J_1/J_2 obtained in the limiting case differs only insignificantly (4 per cent) from the value of J_1/J_2 for the case of a parabolic distribution.

Taking into account that J_1/J_2 is the correction coefficient for the calculation of the influences of transverse shears, it is possible to conclude that, with the accuracy of the initial assumptions of the problem, a reasonably chosen function $f(z)$ will not affect the value of the critical pressure P.

5. As an illustration we consider a numerical example. From the formula for P_*, it is easily seen that the consideration of the transverse shears gives rise to the correction term

$$\delta = \frac{m^2\pi^2}{10} \frac{h^2}{l^2} \frac{E}{G(1-\nu^2)} K$$

depending on the number of half-waves m, the relative thickness h/l, and the physical characteristics of the material E, E', G, ν, and that it can differ significantly from zero.

Let

$$h / l = 0.2, \qquad \nu = 0.5, \qquad E / E' = 1.2, \qquad E / G \sim 3.0$$

Then

$$\delta = 0.138, \quad \text{when } m = 1, \quad \delta = 0.552, \quad \text{when } m = 2$$

Now we note that the consideration of transverse shear when a plate is deformed into one half of a sine-wave gives a correction of the order 13 per cent, and when deformed into two half-waves the correction rises to 50 per cent.

Thus, the disregard of effects connected with transverse shear can in some cases lead to important errors.

The consideration of transverse shear can have still greater influence on the result, since the coefficient K in the dependence on the shape of the curve $\varepsilon = f(\sigma)$ can be greater than unity (in the example treated $K = 0.875$), and the basic physical factor, E/G, characterizing phenomena connected with a consideration of transverse shears can be greater than 3.0.

The author expresses his indebtedness to Iu.N. Rabotnov for

discussion of the preceding work.

BIBLIOGRAPHY

1. Kachanov, L.M., *Osnovy teorii plastichnosti (Foundations of the Theory of Plasticity)*. Gostekhizdat, 1956.

2. Shanley, F.R., Inelastic Column Theory. *Journal of the Aeronautical Sciences*, 14, No. 5, 1947.

3. Rabotnov, Iu.N. and Shesterikov, S.A., Ustoichivost' sterzhnei i plastinok v usloviiakh polzuchesti (Stability of bars and plates under creep conditions). *PMM* Vol. 21, No. 3, 1957.

4. Ambartsumian, S.A., *Teoriia anizotropnykh obolochek (Theory of anisotropic shells)*. Fizmatgiz, 1961.

5. Filin, A.P., K voprosu opredeleniia usilii v staticheski neopredelimykh sistemakh pri nelineinoi zavisimosti mezhdu napriazheniiami i deformatsiiami (On the problem of determining the stresses in statically indeterminate systems with a nonlinear relation between stresses and strains). *Trudy LIIVT*, No. 25, 1958.

6. Kachanov, L.M., *Teoriia polzuchesti (Theory of creep)*. Fizmatgiz, 1960.

7. Ambartsumian, S.A., K teorii izgiba anizotropnykh plastinok (On the theory of flexure of anisotropic plates). *Izv. Akad. Nauk SSSR*, OTD, tekhn. n., No. 5, 1958.

Translated by D.B.McV.